# RHIC-II electron cooling
# based on magnetized approach

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# Outline



1. Theoretical study of magnetized friction force.

   (BNL in collaboration with Tech-X (Colorado) and Dubna (Russia))

   1.1 Simulations with VORPAL code

   1.2 For asymptotic cases – benchmarking with BETACOOL

2. Experimental benchmarking of magnetized friction force.

   (BNL in collaboration with TSL (Sweden) and Dubna (Russia))

3. Cooling dynamics simulation. Parameter of magnetized cooler.

# Magnetized friction force



At a minimum, we want to be sure that we are using the most appropriate and accurate cooling force formulas.

"electron cooling theory is well understood"

1. Infinite magnetic field appoximation
   (Derbenev-Skrinsky (D-S), Derbenev-Skrinsky-Meshkov (D-S-M)).

2. Empiric formula
   (V. Parkhomchuk (VP))
   (any strength of the field) – can show very different cooling dynamics for some parameters. Also, has different numerical factors.

Different formulas agree with one another within factor of 10 – not good for high-energy estimates

# Codes used for Friction Force studies



**We use:**

1. **VORPAL code (Tech-X, Colorado) – uses molecular dynamics techniques to explicitly resolve close binary collisions and thus capture friction and diffusion tensors with a bare minimum of physical assumptions.**

   **C. Nieter, J. Cary, J. Comp. Phys. 196, p. 448 (2004)**

   **D. Bruhwiler et al., AIP Conf. Proc. 773 (Bensheim, 2004), p. 394.**

2. **BETACOOL code (Dubna, Russia)- numerical integration of analytic formulas over electron velocity distribution was added for comparison with simple asymptotic expressions and VORPAL results.**

   **The BETACOOL program, http://lepta.jinr.ru**

# Magnetized friction force
# - approximation of strong magnetic field



**Numerical integration using Derbenev-Skrinsky (D-S) expressions for the magnetized collisions (BETACOOL):**

$$F_{\perp}\left(V_{\perp},V_{\parallel}\right)=-\frac{2\pi Z^{2}e^{4}n_{e}L_{M}}{m}\int\frac{V_{\perp}\left(V_{\perp}^{2}-2\left(V_{\parallel}-v_{e}\right)^{2}\right)}{\left(V_{\perp}^{2}+\left(V_{\parallel}-v_{e}\right)^{2}\right)^{5/2}}f\left(v_{e}\right)dv_{e}$$

$$F_{\parallel}\left(V_{\perp},V_{\parallel}\right)=-\frac{2\pi Z^{2}e^{4}n_{e}}{m}\int\left(L_{M}\frac{3V_{\perp}^{2}\left(V_{\parallel}-v_{e}\right)}{\left(V_{\perp}^{2}+\left(V_{\parallel}-v_{e}\right)^{2}\right)^{5/2}}+2\frac{V_{\parallel}-v_{e}}{\left(V_{\perp}^{2}+\left(V_{\parallel}-v_{e}\right)^{2}\right)^{3/2}}\right)f\left(v_{e}\right)dv_{e}$$

**Asymptotic expressions for all three type of collisions (Derbenev-Skrinsky-Meshkov (D-S-M)):**

$$F_{\perp}\approx-\frac{2\pi Z^{2}e^{4}n_{e}}{m}v_{\perp}\begin{cases}\dfrac{1}{v^{3}}\left(2L_{F}+\dfrac{v_{\perp}^{2}-2v_{\parallel}^{2}}{v^{2}}L_{M}\right),\{I\}\\[2ex]\dfrac{2}{\Delta_{\perp}^{3}}\left(L_{F}+N_{col}L_{A}\right)+\dfrac{v_{\perp}^{2}-2v_{\parallel}^{2}}{v^{2}}\dfrac{L_{M}}{v^{3}},\{II\}\\[2ex]\dfrac{2}{\Delta_{\perp}^{3}}\left(L_{F}+N_{col}L_{A}\right)+\dfrac{L_{M}}{\Delta_{\parallel}^{3}},\{III\}\end{cases}$$

$$F_{\parallel}\approx-\frac{2\pi Z^{2}e^{4}n_{e}}{m}v_{\parallel}\begin{cases}\dfrac{1}{v^{3}}\left(2L_{F}+\dfrac{3v_{\perp}^{2}}{v^{2}}L_{M}+2\right),\{I\}\\[2ex]\dfrac{2}{\Delta_{\perp}^{2}v_{\parallel}}\left(L_{F}+N_{col}L_{A}\right)+\left(\dfrac{3v_{\perp}^{2}}{v^{2}}L_{M}+2\right)\dfrac{1}{v^{3}},\{II_{a}\}\\[2ex]\dfrac{2}{\Delta_{\perp}^{2}\Delta_{\parallel}}\left(L_{F}+N_{col}L_{A}\right)+\dfrac{L_{M}}{\Delta_{\parallel}^{3}},\{II_{b},III\}\end{cases}$$

# Finite magnetic field



**Empiric formula by V. Parkhomchuk (VP) (NIM, 2000):**

$$\vec{F} = -\vec{v}\,\frac{4Z^2 e^4 n_e L_P}{m}\,\frac{1}{\left(v^2+\Delta^2_{e,eff}\right)^{3/2}} \qquad L_P = \ln\left(\frac{\rho_{\max}+\rho_{\min}+\langle\rho_\perp\rangle}{\rho_{\min}+\langle\rho_\perp\rangle}\right)$$

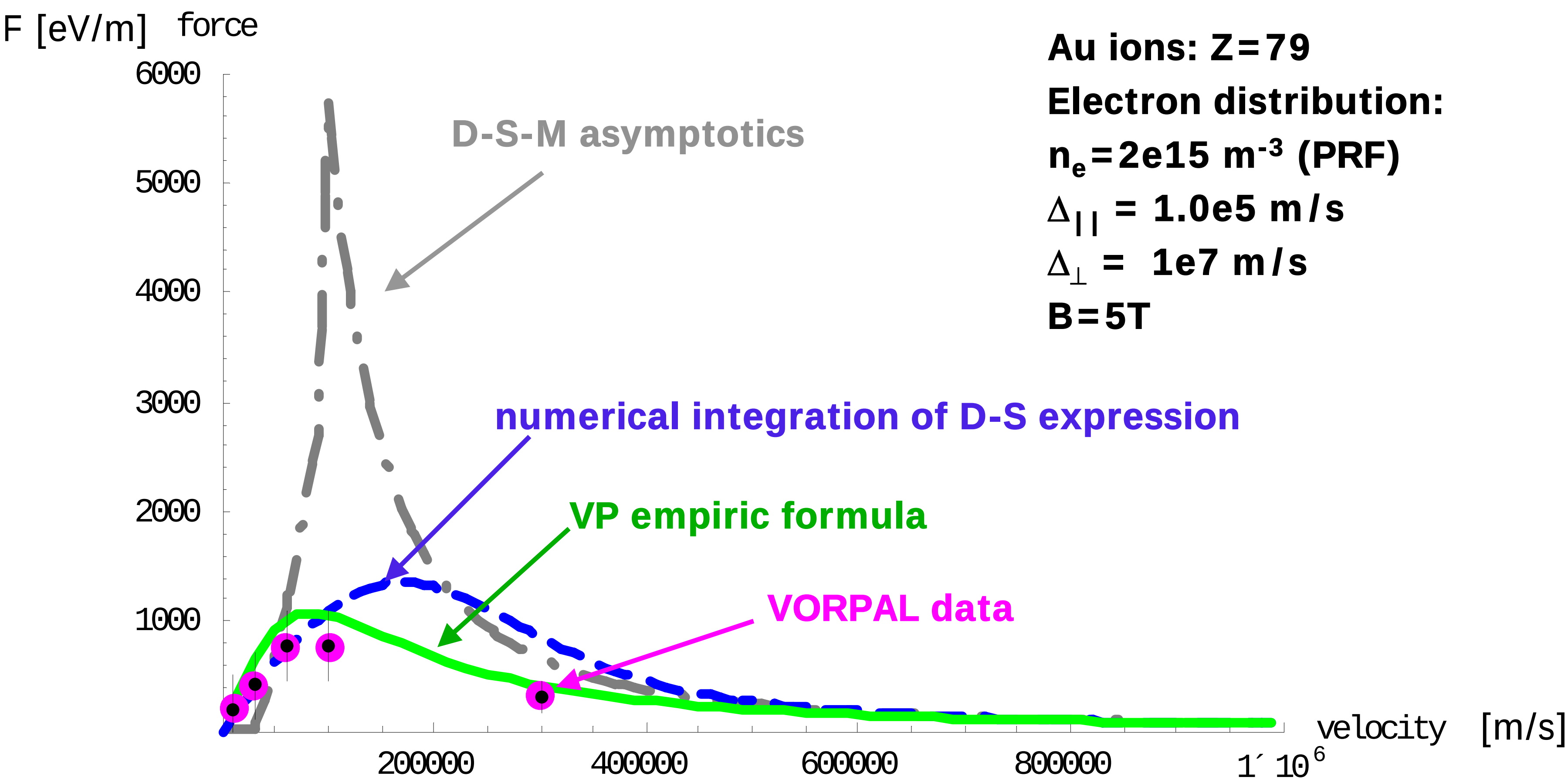
**1. Similar to D-S asymptotics at low velocities $v < \Delta_{||}$**

**2. Very different at large velocities $v >> \Delta_{||}$ - both in numerical factor and dependence on angle with respect to the magnetic field direction.**

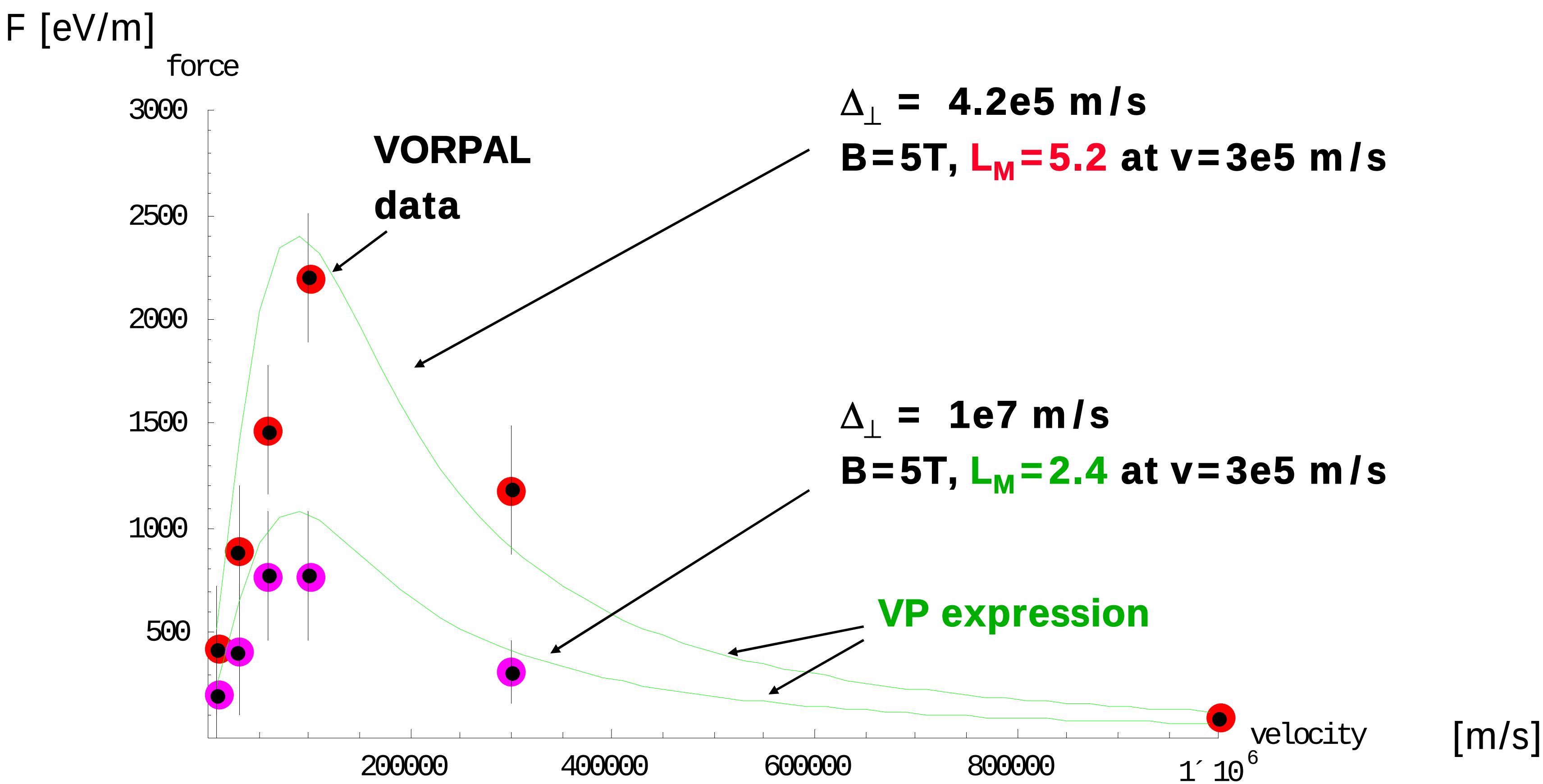
**Systematic studies were done to explore magnetized friction force formulas in various regimes. Some of these studies are reported in the next slides, using parameters of the RHIC-II cooler based on the magnetized approach.**

# Friction force for ion velocity along magnetic field line $V_\perp = 0$





# Friction force for ion velocity along magnetic field line ($V_\perp$ =0) for two different degrees of magnetization



# Angular dependence at large relative velocities



Strong magnetic field results in different expressions for the transverse and longitudinal components of the friction force both of which now depend on both transverse and longitudinal velocity.

But how important is such "angular anisotropy" of the friction force for finite magnetization?

This question was already addressed by Parkhomchuk (NIM, 2000), using simulations with zero temperature electrons.

Using VORPAL, we did careful studies both for zero and finite temperature electrons which allowed us to resolve remaining issues.

# Angular dependence for longitudinal component of the friction force



**Empiric formula by V. Parkhomchuk (VP)**

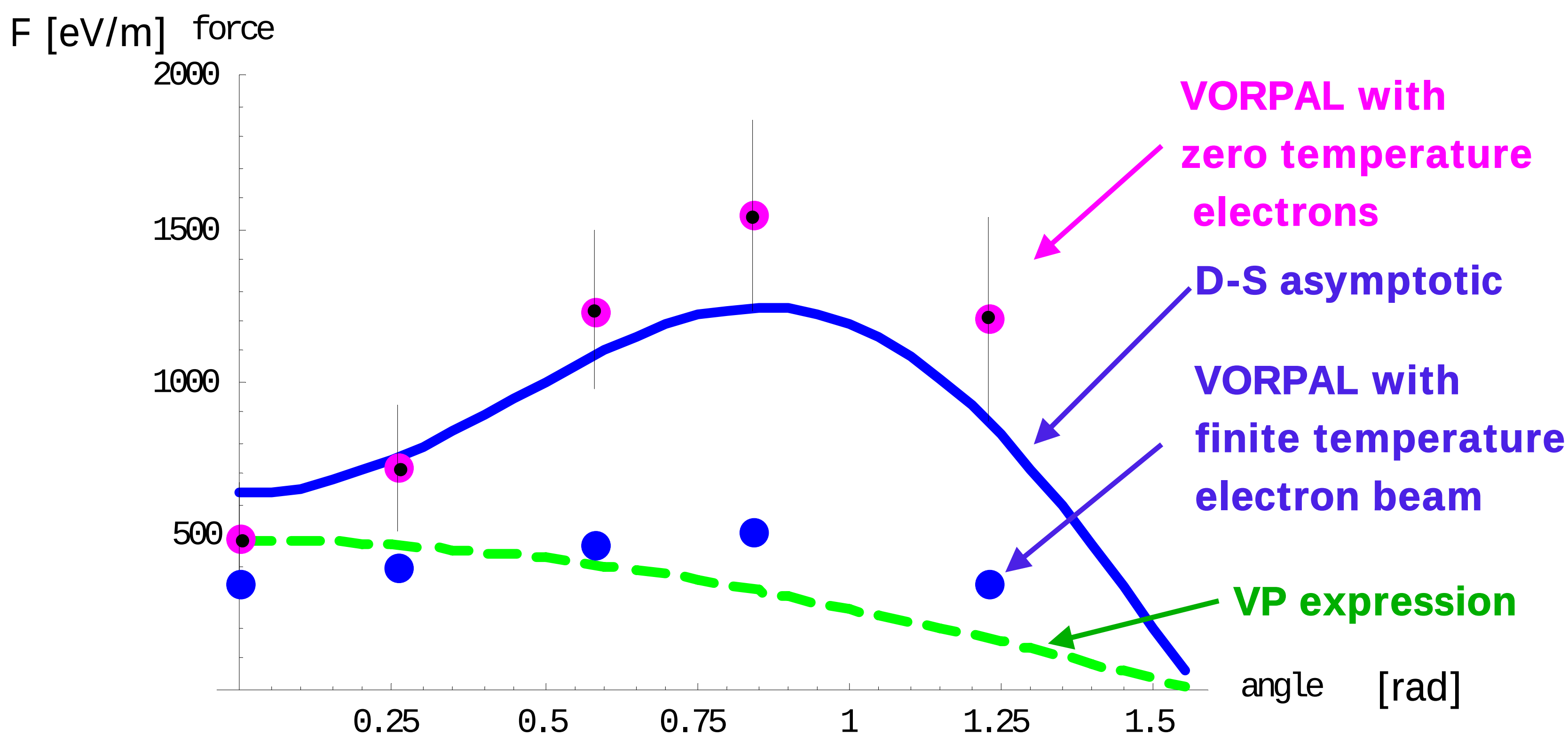
$$\mathbf{F}^{VP} = -\frac{1}{\pi}\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\Lambda^{M}\frac{\mathbf{V}_{ion}}{\left(V_{ion}^{2}+V_{eff}^{2}\right)^{3/2}}$$

**Asymptotic formula by Derbenev-Skrinsky (D-S)**

$$\mathrm{F}_{\parallel}^{DS} = -\frac{3}{2}\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\left[\Lambda^{A}(V_{ion})\left(\frac{V_{\perp}}{V_{ion}}\right)^{2}+\frac{2}{3}\right]\frac{V_{\parallel}}{V_{ion}^{3}}$$

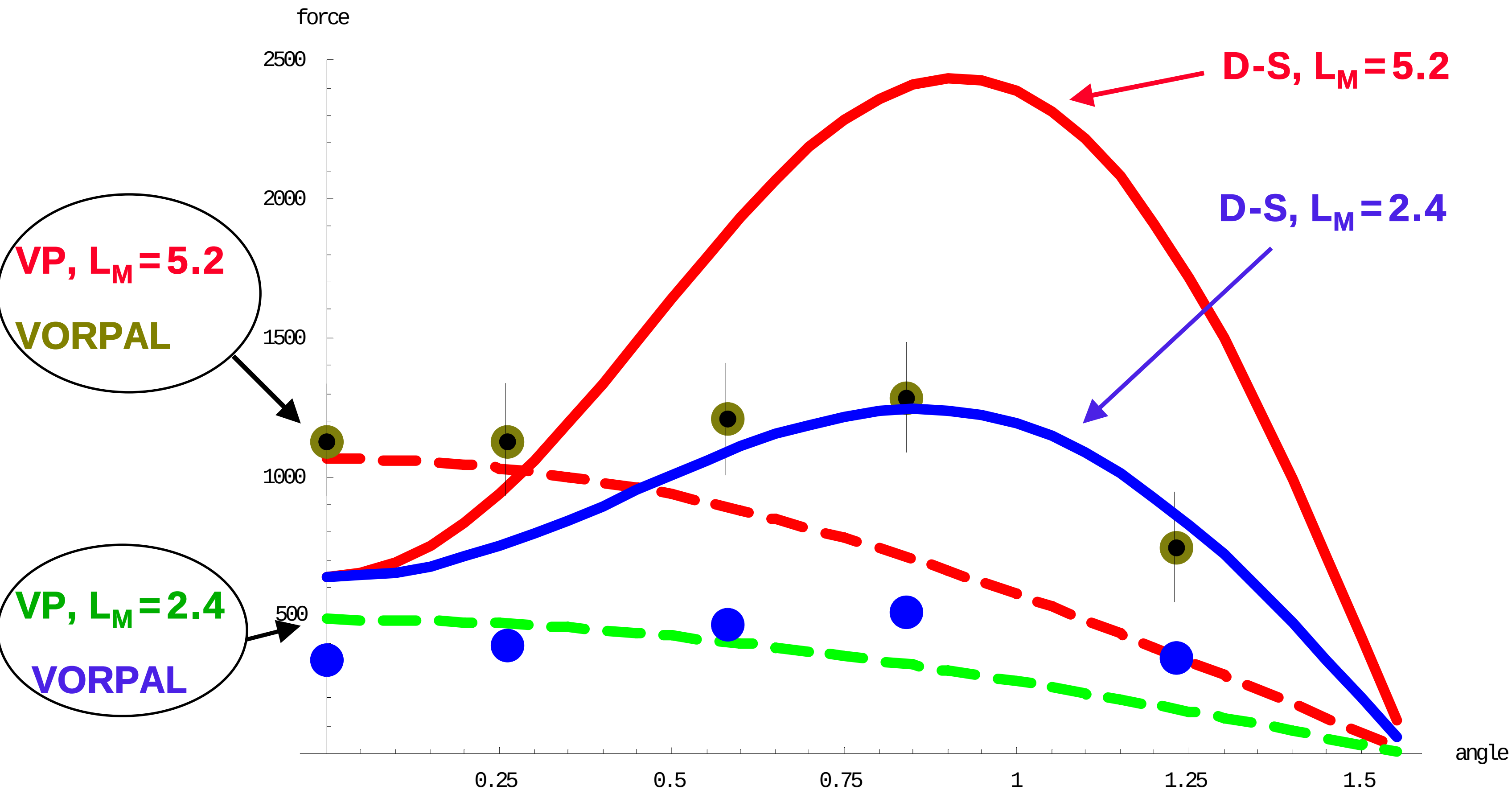
# Angular dependence for $V_{ion}$=3e5 m/s (B=5T, for $\Delta_{ex,y}$=8e6 m/s, $L_M$=2.4)





# Longitudinal friction force – scaling with magnetized logarithm for finite temperature electron beam





# Conclusions on magnetized formulas



1. Comprehensive examination of theoretical formulas was performed.

2. Order of magnitude difference in these models was resolved.

3. Origin of finite friction force at zero transverse angles was explained.

Performed benchmarking enables accurate description of cooling process based on the magnetized approach.

Description within factor of two can be done using numerical integration added to BETACOOL as a result of these studies. If better than factor of 2 accuracy is needed one needs to perform simulations similar to the one done with VORPAL.

# Experimental benchmarking: Major goals

(In collaboration with INTAS grant "Advanced beam dynamics for storage rings", O. Boine-Frankenheim et al.)

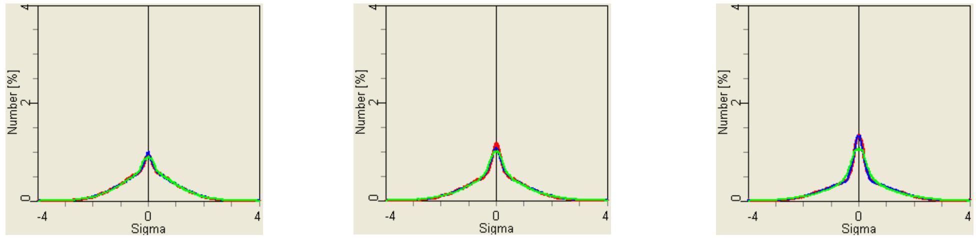


1. With well controlled experiments – systematically study friction force dependence on various parameters such as current, alignment angle, magnetic field.

2. Using low-energy cooler try to reproduce conditions possible at high-energy cooling:

2.1) Different magnetization regimes – possible transition from good to bad magnetization

2.2) Transient cooling – when as a result of slow cooling one first has clear formation of beam core with subsequent cooling of tails – need to benchmark IBS models for such distributions. very important for collider



# Experiments done at Svedberg Laboratory, Uppsala, Sweden, CELSIUS (December 2004, March 2005)

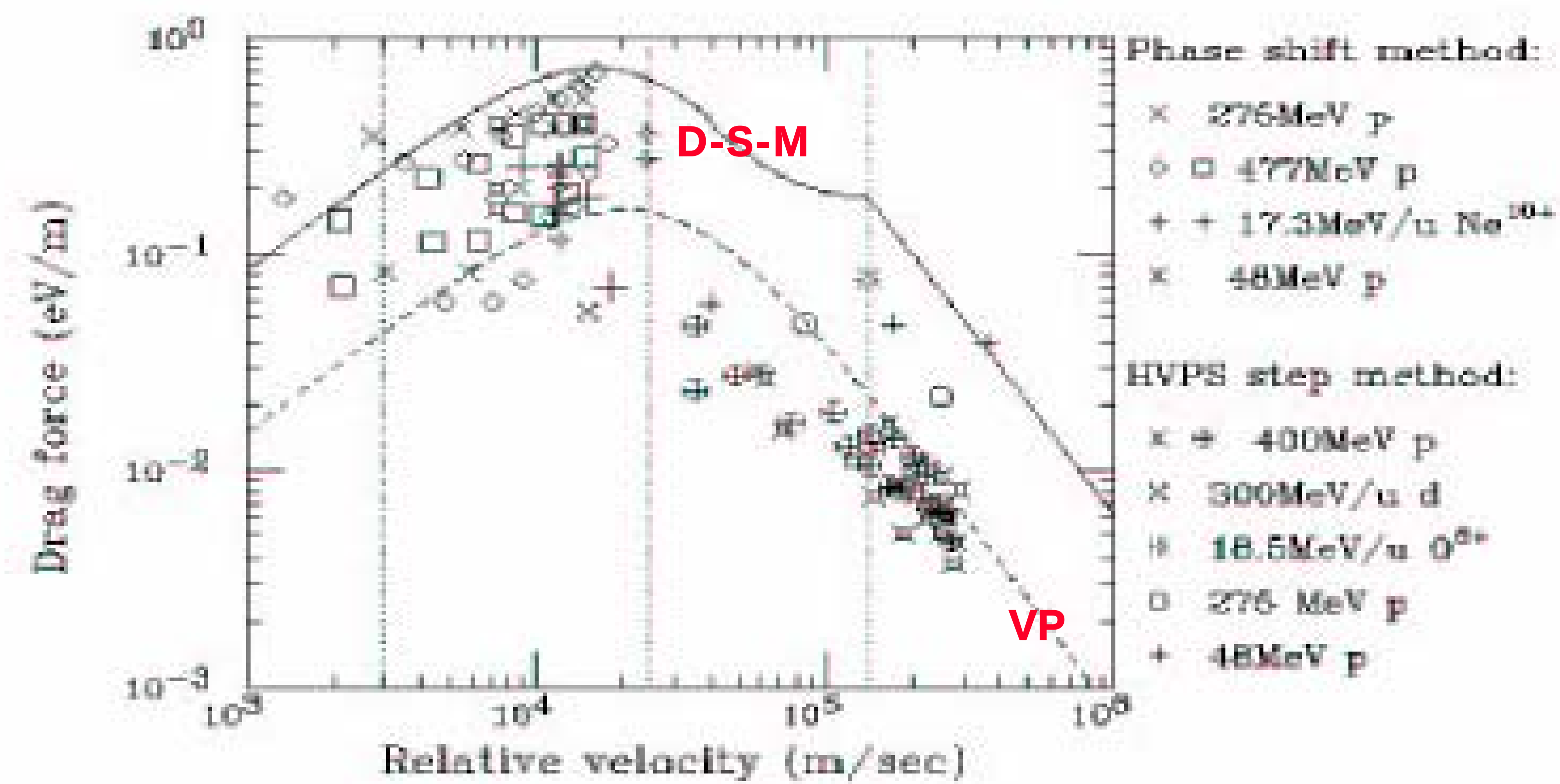
(B.Galnander, T. Lofnes, V. Ziemann (TSL); A. Fedotov, V.Litvinenko (BNL); A. Sidorin, A. Smirnov (Dubna))

1. For standard parameters of the cooler: current dependence
2. Dependence on $V_{effective}$
3. Studies of transient cooling; formation of bi-Gaussian distribution; IBS for non-Gaussian distributions
4. Study for various degrees of magnetization
5. Effect of solenoid errors

# Accuracy of the measurement: Example of some previous comparison of experimental data with D-S-M and VP formulas.



**Y-N. Rao et al.: CELSIUS, Sweden'2001:**



# Accuracy of the measurement



1. One needs to introduce small velocity difference between electrons and ions – typically, voltage step is used to change energy of electrons.

2. One needs accurate measurement of the phase difference between the bunch and RF signal.

In our experiment at CELSIUS:

1. Changing RF frequency – allowed very fine steps in velocity difference (done before, for example, at IUCF).

2. Instead of network analyzer without phase lock loop the phase was measured by phase discriminator.

**As a result, very accurate experimental data was obtained !**

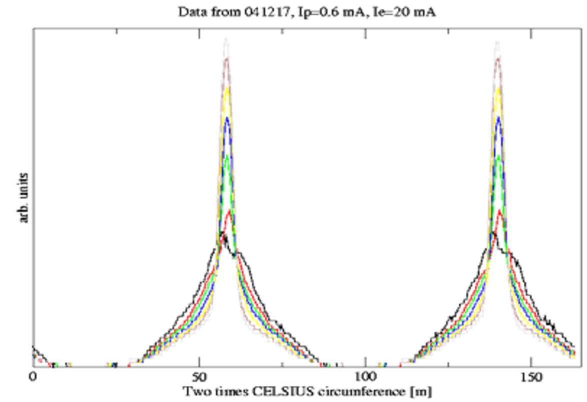
# Example of experiment #3:

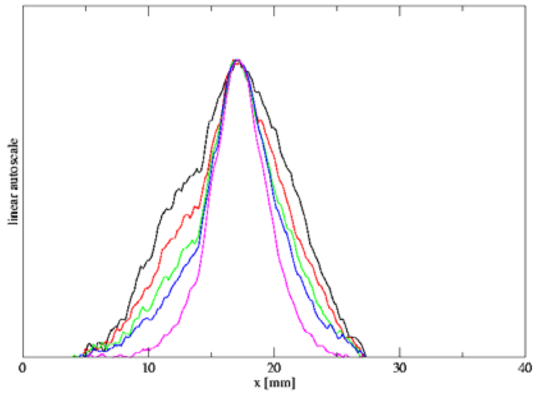


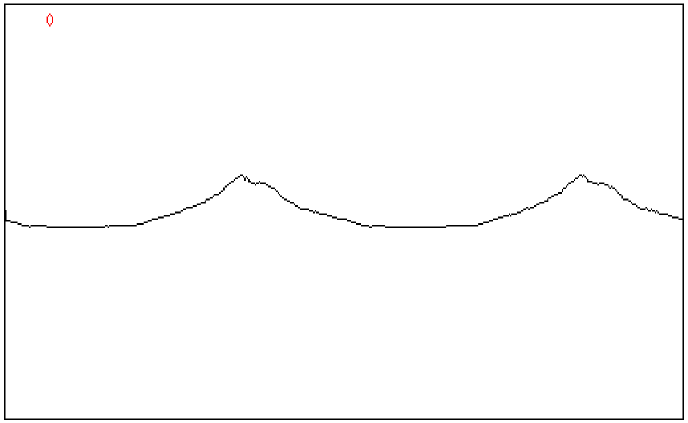
3. Measured "transient cooling" (IBS+COOLING) both for longitudinal and transverse profiles:

Test models of IBS for

non-Gaussian distribution – needed for high-energy cooling.

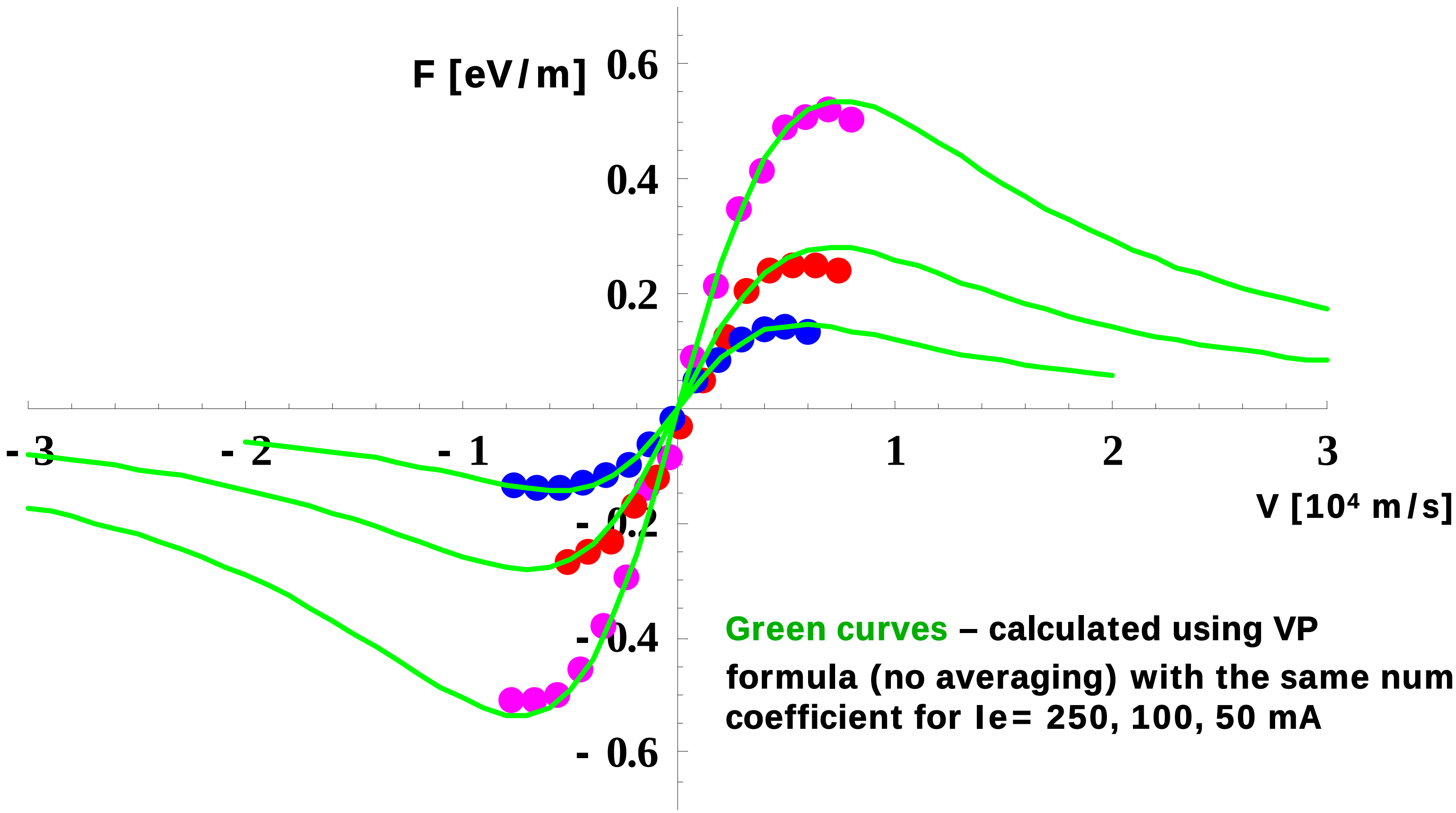


# Example of current dependence studies: B=0.1T, electron current Ie=250 (pink color), 100 (red), 50 (blue) mA






**Green curves – calculated using VP formula (no averaging) with the same numeric coefficient for Ie= 250, 100, 50 mA**

# Observations

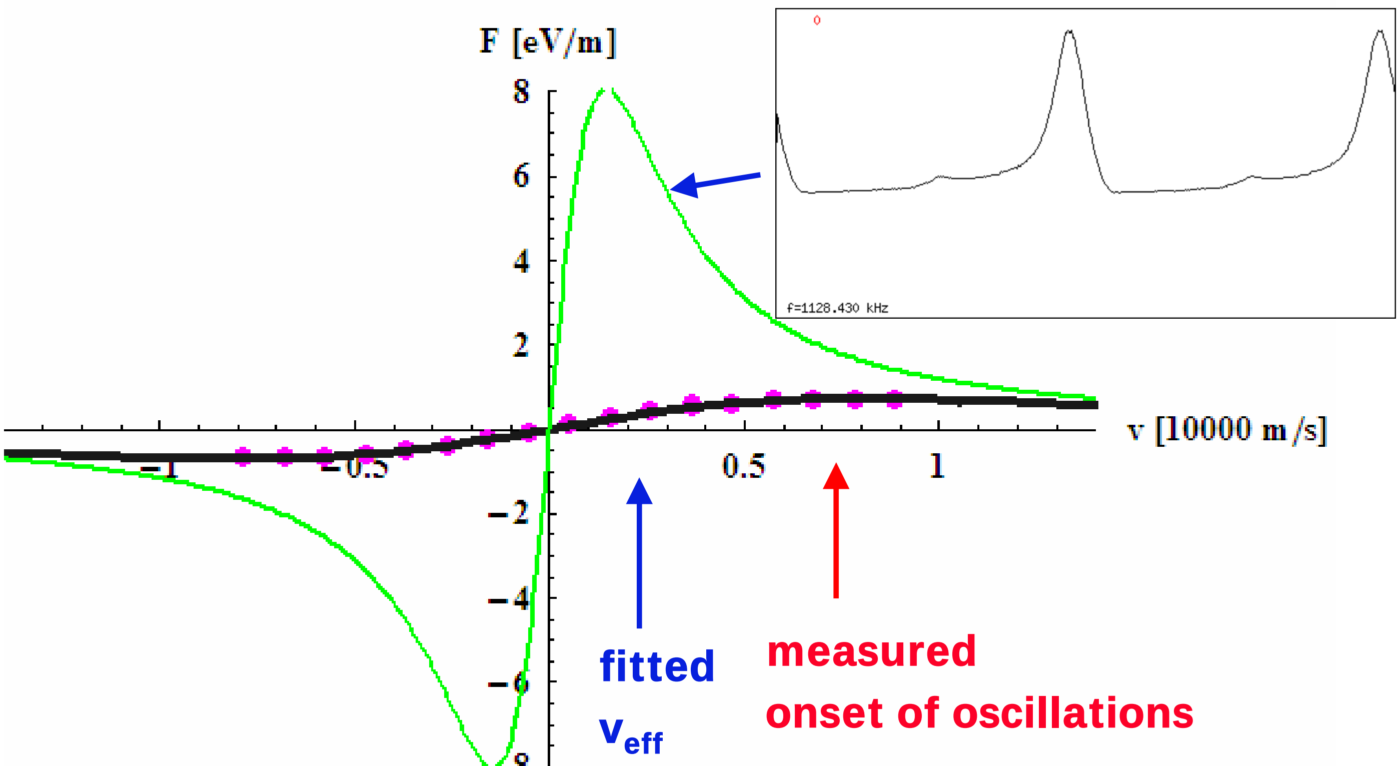


- Using single-particle formula allows to fit experimental data and extract V_effective.
- However, since rms velocity spreads of cooled proton beam are significant (for our measurements, we would need to have dp/p=1e-5 and ε=1e-9 m rad to neglect this effect, while parameter of the proton beam with which we did measurements typically had about dp/p=5e-5 and ε=5e-8 m rad), fitted V_effective has contribution from this effect.

The accurate procedure is then to measure rms velocities of the distribution and average single-particle formulas over the proton distribution.

This was done for all 10's of friction force curves which were measured for various parameters

# First approach – one fitting parameter Veff




**Longitudinal profiles:**

**expected onset of oscillations for small Veff**

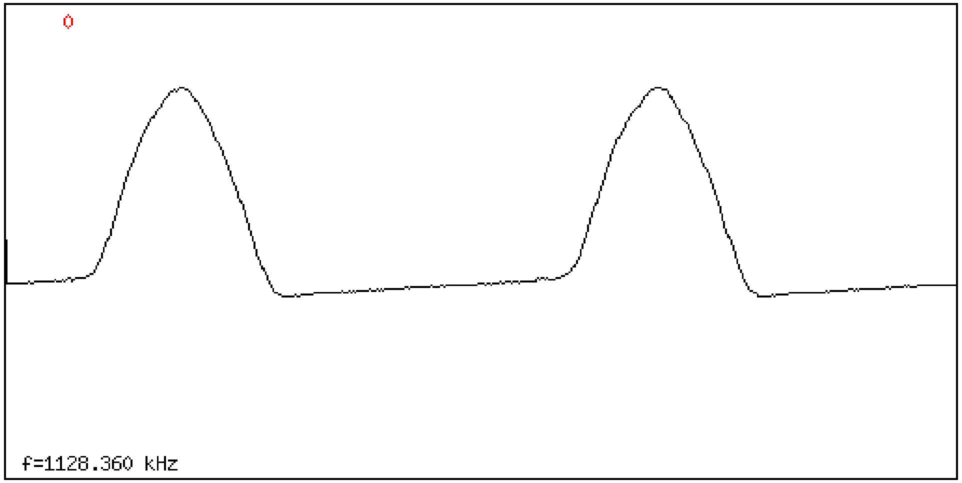
$$\langle F \rangle = C \frac{4\pi Z^2 e^4 n_e}{m\sqrt{2\pi}\,\Delta_\perp^2 \Delta_\parallel} \int_0^\infty \int_{-\infty}^{\infty} \frac{v_\parallel L_M(v_\perp, v_\parallel, v_{eff})}{(v_\perp^2 + v_\parallel^2 + v_{eff}^2)^{3/2}} \exp\left(-\frac{v_\perp^2}{2\Delta_\perp^2} - \frac{(v_\parallel - v_0)^2}{2\Delta_\parallel^2}\right) v_\perp \, dv_\parallel \, dv_\perp$$

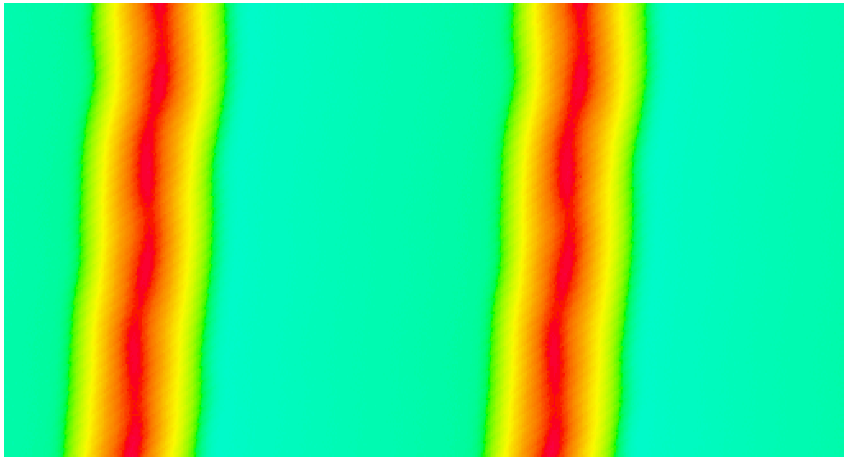
**$C = 1/\pi$**

# Measurements of longitudinal friction force maximum



## Approaching friction force maximum



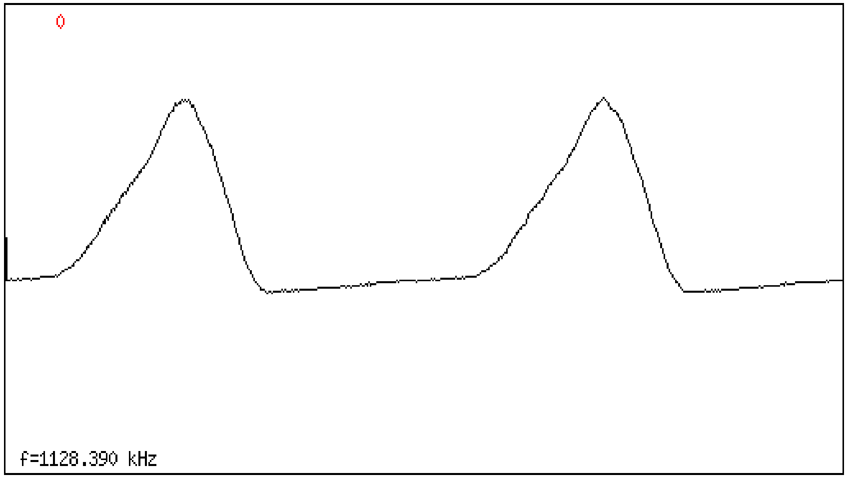



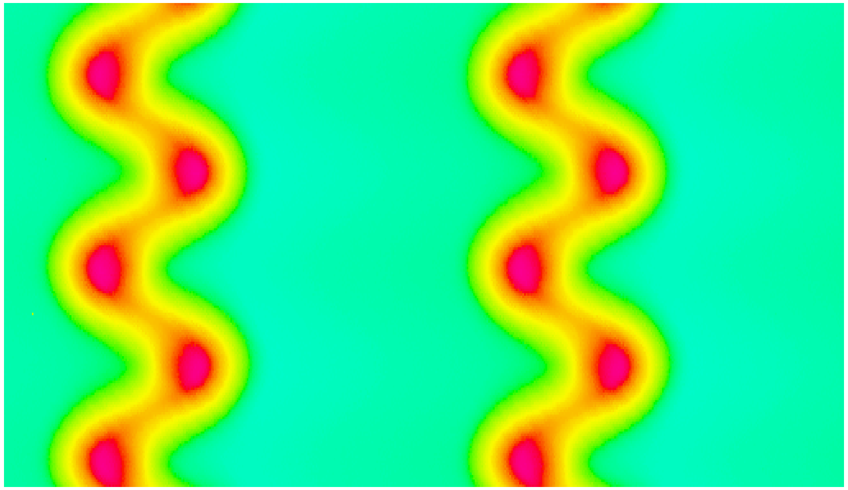
## Longitudinal profiles

# Measurements in non-linear part of the friction force



**far past the maximum**

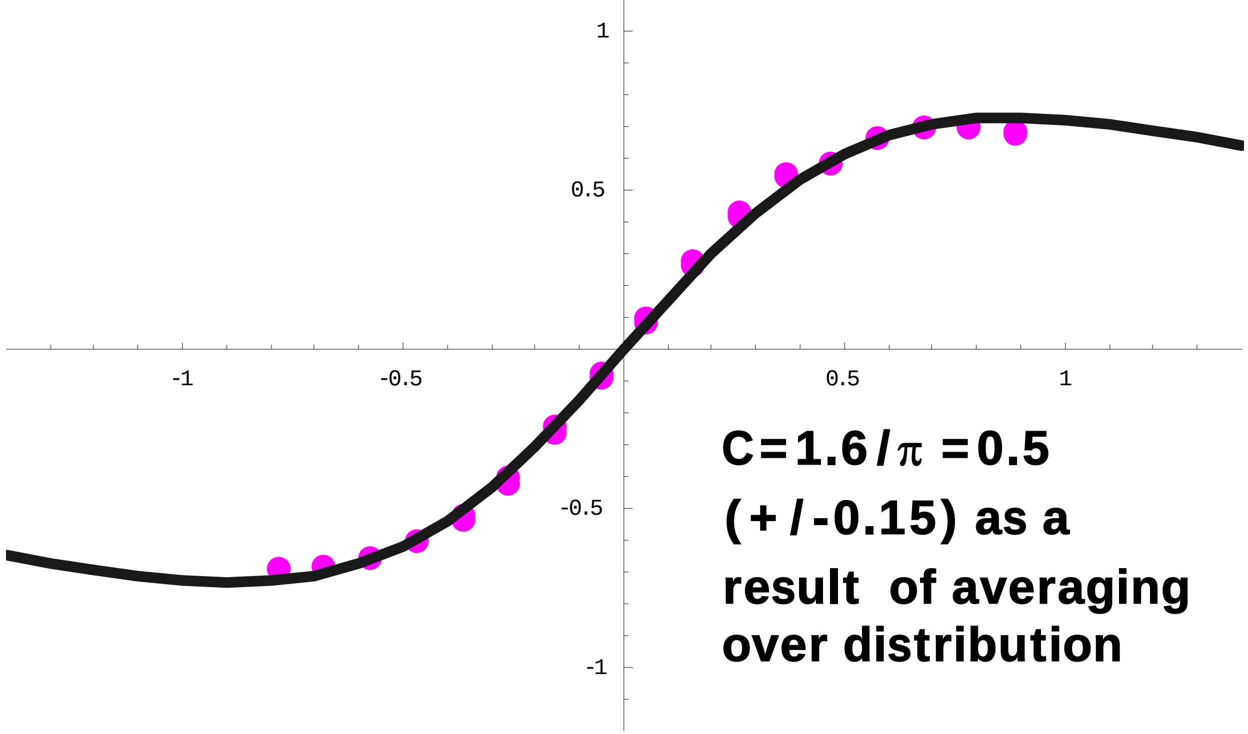


## Second approach - fitting C while Veff is considered known from measurements (taking into account that cooled proton beam is space-charge dominated longitudinally)



$$\langle F \rangle = C \frac{4\pi Z^2 e^4 n_e}{m\sqrt{2\pi}\Delta_\perp^{\,2}\Delta_\parallel} \int_0^\infty \int_{-\infty}^\infty \frac{v_\parallel L_M(v_\perp, v_\parallel, v_{eff})}{(v_\perp^{\,2} + v_\parallel^{\,2} + v_{eff}^{\,2})^{3/2}} \exp\left( -\frac{v_\perp^{\,2}}{2\Delta_\perp^{\,2}} - \frac{(v_\parallel - v_0)^2}{2\Delta_\parallel^{\,2}} \right) v_\perp dv_\parallel dv_\perp$$




**$C = 1.6/\pi = 0.5$ (+/-0.15) as a result of averaging over distribution**

**Compared to single-particle formulas with coefficients of $C_{Derbenev} = 0.4$ and $C_{Parkhomchuk} = 0.32$**

# RHIC e-cooling dynamics simulations



**Au ions at $\gamma$=108:**

**Parameters of Magnetized Cooling:**

**Solenoid:**

length per ring: L=30x2=60m

magnetic field: 2-5T

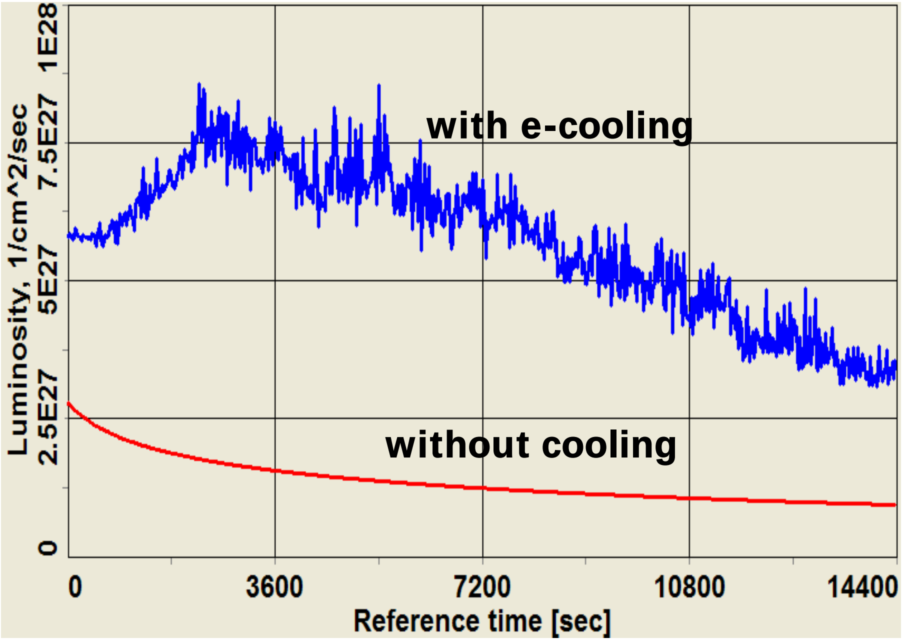
rms imperfections: $1*10^{-5}$

**Electron beam:**

charge q: 20nC

rms emittance $\varepsilon_x$: 50 um

rms momentum spread $\sigma_p$: < 0.001




**Factor of 10 increase in luminosity**

## Summary for magnetized cooling



- **Order of magnitude difference between magnetized friction force formulas was resolved. The force can be now described with uncertainty of factor of 2 or even less, if VORPAL code is used.**
- **Dedicated experiments were performed. Experimental data related to the friction force description was benchmarked.**
- **Parameters of cooler were optimized.**

**Based on detailed study of magnetized friction force (theory/simulations/experiments), confidence in cooling dynamics simulation was significantly improved.**

**Magnetized cooling for RHIC-II with optimized parameters of e-cooler (given on previous slide) is feasible.**